THE TEACHING OF MATHEMATICS: THE NEED AND THE METHOD

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The *sine qua non* in mathematics in *accuray*. Sylvester said "The fine arts are four: Plastic, Music, Lyric and Mathematic," and Dr. Story has recently defined mathematics as "The Art of Exact Thinking.

Never, in the history of the race, was the art of exact thinking more needed than it is today; the world is full of cranks with half-baked theories of government, religiion and morals,, as well as education itself, based on inaccurate deductions from inadequate and false data, influenced by the prejudice of ignorance and the habit of illogical thinking. Moreover, they send forth their products with fanatical vehemence and their command of language is altogether too great.

It is doubtless high time for *real educators* to think and think with accuracy; for what shall it profit a man (and his race) if he gain the whole art of expression and lose the art of forming true ideas; yea, rather, the world is the loser when, by artful expression, false notions can be sent forth clothed in a pleasing and engaging garment of language, as with authority. It were far better that a man be robbed of his power of expression, if that which he wishes to express be false, because of his inability to think with accuracy. I never think of accuracy of expression without recalling a story told by Mr. McElroy in his lecture on "Some Interesting Inaccuracies." "In one of up-state cities in New York State, a prominent citizen died and the editor of the local paper (writing in his best style) wrote, 'The King of Terrors never entered a happier home;' the compositor, looking at the letters inaccurately formed by the editor, set up the type to read 'The King of Tunis, never entered a happier home;' but as such, it could never get by the all-important proofreader, who, being a man of much learning said at once, 'This will never do; the ruler in Tunis is not called a King—he is the Bey of Tunis;' so the obituary went forth, 'The Bey of Tunis never entered a happier home.'" The "cock-sure advice on how and

when to teach mathematics, given by those who never teach it, is probably compounded by some such formula as that which produced "The Bey of Tunis."

The great development in science, in all directions, was made (and is being made) possible by the development of mathematics, which in turn owes its advance from time to time, to the introduction of new and better symbols, so that mathematics is often defined as "symbolic thought;" mathematics is indeed a language, if one defines a language as a means for the expression of ideas; moreover, mathematics is a universal language, being practically the same the world over, and is therefore of more importance than any one language. Mathematics contributes not only to the great material prosperity and commercial activity, but also to the general moral and spiritual uplift by the introduction of breadth of view, depth of thought and sense of truth.

I agree with Myers in his "seventh principle," as stated in the February, 1920, number of the *Mathematics Teacher,* that good high school mathematics and good mathematical teaching are chiefly beneficial to the public school pupil anywhere, because, and in so far as, they beget and foster the habit of taking a rational attitude toward problematic situations; i. e. the habit of basing conclusions on the underlying facts." This, he says, "Is the chief reason why we cannot afford to allow the mathematical element in education to be reduced to the minimum essentials."

Mental discipline is of first importance in education and today we seem to be in danger of losing most of it through a general attitude of relaxing all discipline. The denials of the disciplinary value of mathematical training and the claim that it does not furnish any mental power that can be transferred, have been based on insufficient data, giving negative results, while the fact of increased ability due to mathematical training has impressed itself on keen observers for many centuries: certainly as long ago as the time of Plato and other Greek philosophers, as pointed out by Cajori in his article printed in the "*Mathematics Teacher*" for December, 1920, entitled "Greek Philosphers on the Disciplinary Value of Mathematics," in the course of which he quotes the following statement from Plato's

"*Republic*": "In *all* departments of Knowledge, *as experience proves,* any one who has studied geometry is infinitely quicker of apprehension than one who has not." Note the word, *proves;* when an ancient Greek used the word proves no one can doubt that the facts carried conviction to his mind in no uncertain terms. (Revisionists will please note, also, the word *geometry* used in this connection). Cajori ends his article with the statement "Philosophers of great eminence, like Plato and his disciples placed extraordinary emphasis upon the mind-training value of mathematics."

It must also be borne in mind that it was the study of geometry that first awakened to activity such minds as those of Newton and Einstein.

To me, one of the great contributions of mathematics to mind training (when properly taught) is the formation of good habits of thought and of expression which are the very first essentials for a student in mathematics; habits are presistent and certainly every child must be trained to possess the habit, as Myers says, "of taking a rational attitude toward problematic situations'" with which every citizen will be continually confronted.

There can be no doubt of the need. The great difficulty has been and is likely to be with the methods used in *teaching* mathematics, *especially* in the grades, where the first habits, good or bad, are formed. I feel quite sure that *good teaching* (the best possible) is of more importance in the grades than at any other level ,and is far more difficult work than is necessary in later years, especially if the work is well done in the grades so that correct habits are formed. We ought to insist on thoroughly trained teachers of *unusual* ability for the grade schools and pay them what they are worth—probably more than is paid to teachers in high schools, or at least as much— since their training would require as much time and effort as that expended by the high school teacher.

If the present methods of some so-called educators prevail the "viscious circle" will soon bring us to the lowest depth, if indeed we have not almost reached that level now in the grades:— First of all many pupils now get through the grades with very little mathematics—a very little working knowledge and no comprehension of the fundamental processes; second, the so-called

educator says "Well its too bad to burden these promising citizens with such stuff as algebra and geometry; these are especially annoying and burdensome to girls who have no natural ability in this direction anyway, so it will be better not to require them to take mathematics, or at least not very much;" third, today practically all of our teachers for the grades come from these same high school girls, after they have had a "Normal School training" where, however, they are taught methods of teaching but not the things to be taught; so they begin their teaching of mathematics with less knowledge than their former teachers and the downward march continues.

In the days of "long ago" when Normal Schools were first invented (at least in one with which I was acquainted in the 80's) the substance of the subjects was thoroughly given by well trained teachers and it was thought essential to know the subject before trying to teach it to some one else.

In many other professions and trades, essential to the life of the community it is necessary that those who enter them shall have the habit of performing the elementary operations of mathematics with *accuracy;* e. g. in the past few months I have been informed by one who for many years had charge of the training of nurses at a well known hospital, that the girls who came for the training could not, in most cases, handle simple fractions, such as ½ of ¼, so essential in their work, or to get a dose of 1-120 of a grain when only 1-60 of a grain tablets were available—they were just as likely to say "give two tablets" as to say "divide the tablet into two parts."

I wonder how many of us are willing to run the risk of being given four times the dose ordered by the doctor, in a serious case.

As teachers of mathematics and as good citizens, we ought to *insist* in some way—yea, in every way—that *all* teachers who are to be intrusted with the teaching of mathematics in the public schools *shall have, at sometime, a thorough course in the subject matter under a skilled teacher with a vigorous insistence on accuracy and the proper use of symbols. Why* should anyone be permitted to teach mathematics who has no more regard for symbols than to teach her pupils (or, at least, allow them) to write $100 - 10 = 90 - 10 = 80 - 10 =$ etc.," until finally $100 = 0$? Yet this is not only done in our schools, but judging

by the work of the freshman in college, this and similar flagrant misuses of symbols are tolerated or may be even taught, not only in the grades but in the high schools.

The proper use of symbols is the very *soul* of mathematics and must be insisted upon from the first grade. The lack of a proper set of symbols to represent numbers, kept the Greeks from making an advance in methods of computation and algebraic analysis, at all comparable to their great work in geometry.

Mathematics has nothing to do with the subject matter involved; its concern is with the operations performed and may be applied to any subject provided its symbols can be given a meaning in that subject. A child must be taught that as soon as a problem has been stated in mathematical form, whether in symbols or arithmetic or algebra, or what not, then he may proceed with the allowable mathematical operations regardless of the things for which these symbols stand; only in the final interpretation of results must he pay attention to the things represented by the symbols. (It is not necessary, for example, for him to think of a ferry-boat transporting a number of cows from one side of the river to the other, when he wishes to transpose a term in x which at the outset was used to represent so many cows). To be sure, close connection with concrete things must be maintained at the beginning, for the child must use "pegs" in his mind on which to hang new ideas. The utility of mathematics must ever be kept before the pupil so that he may always be eager to learn the symbols and operations in order to profit by their use, and it must be made plain to him that his operations with symbols will be made easier and more effective if he is able for the time to forget (or disregard) the things for which the symbols stand.

Instead of a move to shorten the required course in mathematics there should be a tremendous move to lengthen the course, changing its form and content. Unlike the "Dead Languages" with which it was formerly closely and honorably associated, mathematics is forever very much alive; it serves not only in the capacity of scout and guide in pointing the telescope and microscope but it must be relied upon as the heavy artillery in the final attack. As a living, growing art, the ways and means of its presentation must be subject to change. In the last twenty

years, many changes have been made in the mathematical curricula of England, France and Germany. France has *extended* the minimum of mathematics required of all students in the Lycees, to the "elements of calculus" (where it should be). What have the changes done for us, in America? How shall we meet the vital assault now being made to minimize the usefullness of this great instrument for service in the field of education and in every walk of life and how shall we utilize to the best advantage the time now allotted to mathematics in the curriculum?

We, of the colleges, complain that our pupils come to us scarcely half prepared and the teachers of high schools say that they can do no better because their pupils come from the grades with no ability to use the four fundamental processes or to solve simple problems. Let the grade teachers speak for themselves. A few years ago I sent out a "questionaire," and from about 150 replies, from all parts of the United States, I extract the following information, as a kind of summary, from the answers to the two questions on "Difficulties encountered" and "Reforms suggested."

1ST GRADE

Field too large—too much abstract work and not enough work with objects.

2ND GRADE

Work too hard for the child at this age; too much work with abstract numbers. Reforms suggested are: less work for this grade so that better work can be done—leave out written problems and give more drill on "combination and separation" of numbers (addition and subtraction of small numbers).

3RD GRADE

Difficulties:—Inaccuracy—lack of power to think:—rush for answer without thinking of a method of solution;—lack of knowledge of combinations of numbers to 20;—lack of drill in lower grades. *Reforms*—more time for drill in fundamental processes and less variety of subjects;—more mental work and more thorough work in lower grade.

4TH GRADE

Difficulties:—Poor in numbers;—lack knowledge of multiplication table. *Reforms*:—more drill on tables and more practical problems;—teach pupil to test his work.

5TH GRADE

Difficulties;—Inaccuracy the main stumbling block throughout;—deficient in fundamental processes.

Reforms;—Lessen the field of work to get *accuracy* and drill;—confine work in lower grades to the four fundamental processes. Problem should be pictured by pupil before solution is attempted.

7TH GRADE

Difficulties;—*Inaccuracy* the chief fault; deficient in reading and writing and the four fundamentals;—*carelessness;*—children immature for this work.

Reforms;—More mental work;—do not teach a smattering of everything in each grade;—obtain better grounding in fundamentals;—have problems with simple relations;—insist on thoroughness and accuracy.

8TH GRADE

Difficulties;—Inaccuracy;—lack ability in mental calculations and ability to interpret problems;—too many subjects covered in lower grades.

Reforms;—Abolish spiral system;—omit impractical problems with large numbers;—teach more clearly the relation of numbers.

While some of the answers were a little queer (one teacher saying she "loves to teach abstract work because its tangible" and another "they have been well taught but are very inaccurate") nearly all answers were to the point and the one distinct cry, uttered by 90% of the whole lot above the 2nd grade, is "*inaccuracy.*" ("So say we all of us"). It seems quite conclusive that too much is attempted in the lower grades and that accuracy there is sacrificed for speed, which is a fatal mistake. "Safety-first" in mathematics means "accuracy first" and speed aftewards. A great deal of complaint about "Problems too hard" and "too much work in this grade," would be removed in the upper grades if accuracy was insited upon at every step and no advance granted without it (at least 90% accuracy in fundamental operations, and computations).

So, here we are, at different levels, each looking to the one below and shouting for better work and more of it, and often so

loudly that we cannot hear the cry to us from the level above. Will we make any substantial progress and meet successfully the present assault unless we study the whole situation from the bottom upwards? Can the present "National Committee" finish its work on Secondary Schools without a look at its foundation? Let us "slide down the bannister" together or take a quick elevator for the bottom and then make our way cautiously and laboriously back, lending a hand to the workers at each level, if possible, by learning their difficulties and all pulling together to remove the obstructions to progress, if any, imposed by Supervisors, Superintendents, and Boards or by an ignorant public opinion.